## WRITTEN ASSIGNMENT #3 - Solution

1. (4 points) Find a cubic function  $f(x) = ax^3 + bx^2 + cx + d$  whose graph has horizontal tangents at the points (-2, 6) and (2, 0).

**Solution:** Note that the derivative of f(x) with respect to x is

$$f'(x) = 3ax^2 + 2bx + c,$$

since we have 4 unknown parameters then we need 4 equation 2 of which are coming from the f(x) and the other 2 from the f'(x):

$$\begin{cases} a(-2)^3 + b(-2)^2 + c(-2) + d = 6\\ a(2)^3 + b(2)^2 + c(2) + d = 0\\ 3a(-2)^2 + 2b(-2) + c = 0 \end{cases} = \begin{cases} -8a + 4b - 2c + d = 6\\ 8a + 4b + 2c + d = 0\\ 12a - 4b + c = 0\\ 12a + 4b + c = 0 \end{cases}$$

Using your favorite method, we can solve the above system of equations and get that

$$a = \frac{3}{16}, b = 0, c = -\frac{9}{4}, and d = 3.$$

And the cubic function f(x) has the following form:

$$f(x) = \frac{3}{16}x^3 - \frac{9}{4}x + 3$$

2. An object with weight W is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle  $\theta$  with the plane, then the magnitude of the force is

$$F = \frac{\mu W}{\mu \sin \theta + \cos \theta}$$

where  $\mu$  is a constant called the coefficient of friction.

(a) (2 points) Find the rate of change of F with respect to  $\theta$ .

Solution: Using quotient rule, we get

$$\frac{dF}{d\theta} = \frac{0(\mu\sin\theta + \cos\theta) - \mu W(\mu\cos\theta - \sin\theta)}{(\mu\sin\theta + \cos\theta)^2} = \boxed{\frac{\mu W(\sin\theta - \mu\cos\theta)}{(\mu\sin\theta + \cos\theta)^2}}$$

(b) (1 point) When is this rate of change equal to 0?

**Solution:** The rate of change of F equals to 0, when either  $\mu = 0$  or W = 0 or  $\sin \theta = \mu \cos \theta$ , i.e.  $\mu = \tan \theta$  or  $\theta = \tan^{-1}(\mu) + \pi k$  for any  $k \in \mathbb{Z}$ .

(c) (1 point) If W = 50 lb and  $\mu = 0.6$ , draw the graph of F as a function of  $\theta$  and use it to locate the value of  $\theta$  for which  $\frac{dF}{d\theta} = 0$ . Is the value consistent with your answer to part (b)?

